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Determination of residual shear stresses in composites by a modified layer-removal method

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The layer-removal method is often used for measurement of internal stresses in homogeneous polymeric materials. In order to extend the use of the method to laminated composites containing shear stresses, certain refinements are needed. These include (i) determination of twisting moments, (ii) use of varying material properties (elastic moduli) through the thickness of the composite plate, (iii) use of geometric non-linear analysis accounting for large deformations, and (iv) measurement not only of normal curvatures but also twisting curvatures and normal and shear strains. These refinements are necessary, because a non-symmetric laminate is created when layers are removed, which shows large (twisting) curvatures. The modified layer-removal method was theoretically validated on a typical compression-moulded continuous fibre laminate (polyetherimide/glass) and a typical injection-moulded short fibre-reinforced laminate (polycarbonate/glass). The modified method produced good results and the need to use the modified layer-removal analysis is clearly demonstrated.

1. Introduction

Plastics reinforced with continuous and short fibres, namely composite materials, are used increasingly in non-structural as well as structural applications. Almost every composite product produced above room temperature and used at room temperature, contains residual stresses. The major cause for these residual stresses is the difference in thermal expansion coefficient between fibre and matrix in combination with a high processing temperature and a varying fibre orientation through the thickness of a product. These stresses are referred to as thermal anisotropy stresses. Many cases are reported in the literature where these stresses already use up more than half of the matrix strength [1, 2]; they can be normal stresses and shear stresses. For laminated composites with fibres parallel and transverse to the global axis of the product, the thermal anisotropy stresses will be normal stresses. However, often fibres are aligned at different layers to the global axis of the product. As a consequence, the product will also contain thermal anisotropy shear stresses. These residual shear stresses directly load the matrix material, which is the weakest constituent.

The present research deals specifically with the residual shear stresses. Often the Classical Lamination Theory (CLT) is used to predict these stresses. However, there is a lack of experimental validation proving the prediction to be correct. For this validation, a reliable technique is needed.

An established method for experimental evaluation of residual stresses in homogeneous isotropic materials is the layer-removal method [3]. However, the layer-removal method in its original published form is used for the measurement of normal stresses and not shear stresses. Therefore modifications are necessary.

In addition, it cannot be used on composite materials with variation of properties through the thickness. White and Paterson [4, 5] developed the layer-removal method for use on plates with a depth-varying modulus. But, in their final analysis, they did not consider the fact that in composite products the Poisson's ratios can also vary considerably through the thickness. Next, the curvatures in composite plates after layer removal can become large. In that case, CLT no longer provides the correct basis for calculations. Hyer [6, 7], Jun and Hong [8, 9] and Peeters *et al.* [10] showed that large thermal curvatures in asymmetric composite plates cannot be calculated by CLT and that an energy approach can be used.

In this paper, a modified layer removal method is reported for the measurement of residual shear stresses in which variation of mechanical properties through the thickness is taken into account, and in which an energy approach is used to account for the large curvatures resulting from successive layer-removal. The derivations are performed for angle-ply laminates, which typically contain residual shear stresses but can also contain normal stresses.

The modified layer-removal method is theoretically validated for a polyetherimide (PEI) continuous glass fibre-reinforced laminate and polycarbonate (PC) short glass fibre-reinforced laminate. A subsequent paper will show that practical experiments are in good agreement with the theory developed here.

2. Layer-removal method

In the layer-removal method, thin layers are machined from one surface of a plate and the curvature that is produced to restore force equilibrium is measured at each incremental removal. The curvature profile against thickness removed can then be used to derive the stress profile through the thickness in the original plate. The layer-removal analysis can be divided into two parts. The first part deals with the relation between the deformations after layer-removal and the induced force and moment resultants. This part is material- and sample-geometry-dependent. The second part is the relation between the induced moment resultants, M_i, and the stresses, σ_i, in the original sample. This part is independent of material or sample geometry and was derived by Treuting and Read [3]. They performed the derivation for normal stresses in x- or y-directions. However, the same derivation can also be performed for shear stresses. When a layer containing shear is machined from a plate, a shear force and twisting moment resultant are induced on the remaining plate. The shear force, N_{xy}, and twisting moment, M_{xy}, resultants per unit width are given in terms of the shear stress, σ_{xy} or τ_{xy}, present in the laminate, by

$$N_{xy}(z_1) = \int_{-z_0}^{z_1} \tau_{xy}(z)\,\mathrm{d}z \tag{1a}$$

and

$$M_{xy}(z_1) = \int_{-z_0}^{z_1} \tau_{xy}(z)\left(z + \frac{z_0 - z_1}{2}\right)\mathrm{d}z \tag{1b}$$

Similar to the derivation by Treuting and Read [3], this can be transformed to

$$\tau_{xy}(z_1) = \frac{2}{z_0 + z_1} + \frac{\mathrm{d}M_{xy}(z_1)}{dz_1} + \frac{2M_{xy}(z_1)}{(z_0 + z_1)^2} - 4\int_{z_1}^{z_0} \frac{M_{xy}(z)}{(z_0 + z)^3}\,\mathrm{d}z \tag{2}$$

The co-ordinates used are defined in Fig. 1.

The relation between the induced twisting moment resultant, M_{xy}, and the deformations after layer removal for composite plates, will be derived in the next section.

3. Theoretical analysis

The CLT [11] provides a relation between moment resultants per unit width, M_i, and midplane strains, ε_i^0, and curvatures, κ_i, for an angle-ply laminate. The twisting moment resultant per unit width is given by

$$M_{xy} = B_{61}\varepsilon_x^0 + B_{62}\varepsilon_y^0 + B_{63}\varepsilon_{xy}^0 + D_{61}\kappa_x + D_{62}\kappa_y + D_{66}\kappa_{xy} \tag{3}$$

where B_{61}, B_{62}, B_{63}, D_{61}, D_{62} and D_{66} are stiffness coefficients as defined in the CLT. The body co-ordinates definition of Fig. 2 is used. Using Equation 3, the variation in mechanical properties through the thickness is accounted for, and six deformation components have to be measured in order to calculate the twisting moment, M_{xy}.

However, the CLT is strictly valid for small deformations only. A different approach is needed if the curvatures after layer removal become large. Non-linear terms in the strain–displacement relations can no longer be neglected, as is done in the CLT. Accounting for these terms in angle-ply laminates is done using an energy approach described by Jun and Hong [9] and Peeters *et al.* [10]. The Rayleigh–Ritz method [12] is used to obtain solutions. In this approach, the stored potential energy in the system is calculated as a function of unknown Ritz-coefficients, r_i, from an assumed displacement field. This system is in a stable equilibrium when the potential energy is at a minimum. This minimum can be found by minimizing the potential energy, U, according to the

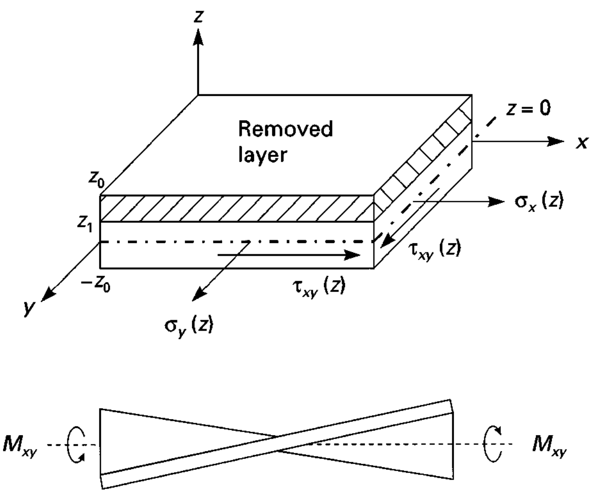

Figure 1 The co-ordinates system used in layer-removal analysis.

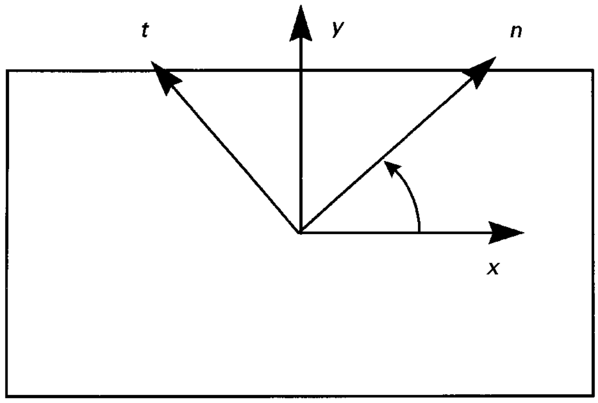

Figure 2 Body (x, y) and principal (n, t) axes.

Ritz-coefficients, r_i

$$\mathrm{d}U = \sum_i \left(\frac{\partial U}{\partial r_i}\right)\partial r_i = 0 \tag{4}$$

This is termed the minimum potential energy (MPE) approach.

The assumed displacement field for an angle-ply laminate has to include terms for normal as well as twisting curvatures. However, the value for the twisting curvature will be zero if the curvatures in the natural body axes x, y and z are rotated to the principal axes of curvature n, t and z (Fig. 2). This is the same approach as used for stresses, where the stresses in any arbitrary direction can be rotated into the principal stress directions with zero shear stress. Hyer [6] uses Mohr's circle to illustrate the meaning of principal curvature. A displacement field in the principal curvature directions n and t is now used to describe the displacements. This displacement field is given by Peeters *et al.* [10]

$$u(n,t) = n\left(a_1 - \frac{c^2}{2} - \frac{ac}{2}n - \frac{a^2}{6}n^2 + a_3 t^2\right)$$
$$v(n,t) = t\left(b_1 - \frac{d^2}{2} - \frac{bd}{2}t - \frac{b^2}{6}t^2 + b_3 n^2\right)$$
$$w(n,t) = \tfrac{1}{2}(an^2 + bt^2 + 2cn + 2dt) \tag{5}$$

where a, a_1, a_3, b, b_1, b_3, c and d are the unknown Ritz-coefficients earlier indicated by r_i. Here a and b are, respectively, the curvatures κ_n and κ_t, and a_1 and b_1 are the midplane strains ε_n^0 and ε_t^0 at $n = t = 0$. This displacement field can be substituted in strain-displacement relations to yield the strains in terms of the unknown Ritz-coefficients, r_i.

A modified von Karman approximation to Greene's strains [8] gives the following strain–displacement relations, in which the right-hand side terms in parentheses are the non-linear terms which are neglected in the CLT

$$\varepsilon_n(z) = \frac{\mathrm{d}u}{\mathrm{d}n} + \frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}n}\right)^2 + z\frac{\mathrm{d}^2 w}{\mathrm{d}n^2}$$
$$\varepsilon_t(z) = \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)^2 + z\frac{\mathrm{d}^2 w}{\mathrm{d}t^2}$$
$$\varepsilon_{nt}(z) = \frac{\mathrm{d}u}{\mathrm{d}^t} + \frac{\mathrm{d}v}{\mathrm{d}n} + \left(\frac{\mathrm{d}w}{\mathrm{d}n}\frac{\mathrm{d}w}{\mathrm{d}t}\right) + 2z\frac{\mathrm{d}^2 w}{\mathrm{d}n\,\mathrm{d}t} \tag{6}$$

Here u, v and w are the displacements in n-, t- and z-directions, ε_n and ε_t the strains in n- and t-directions and ε_{nt} is twice the shear strain as normally defined. This means that ε_{nt} is γ_{nt} which is always used in the CLT.

The potential energy in a plane stress situation, of a plate from which part is machined off, is given by (in contracted notation and using the summation convention)

$$U = \int_V (\tfrac{1}{2}Q_{ij}^{*}\varepsilon_i - \varepsilon_j - \sigma_j\varepsilon_j)\,\mathrm{d}V \;\text{ with }\; i,j = n,t,nt \tag{7}$$

in which σ_j are the stresses in the sample *before* layer removal, ε_i and ε_j are the strains in the sample *after* layer removal and Q_{ij}^{*} are the transformed stiffness coefficients for a unidirectional ply as defined in the CLT. The potential energy is now expressed in terms of the n, t co-ordinate system, indicated in Fig. 2.

Carrying out the integration over the thickness in Equation 7 and using $\varepsilon_i(z) = \varepsilon_i^0 + z\,\kappa_i$ gives

$$U = \iint (\tfrac{1}{2}A_{ij}\varepsilon_i\varepsilon_j^0 + B_{ij}\varepsilon_i^0\kappa_j + \tfrac{1}{2}D_{ij}\kappa_i\kappa_j - N_j\varepsilon_j^0$$
$$- M_j\kappa_j)\,\mathrm{d}n\,\mathrm{d}t \;\text{ with }\; i,j = n,t,nt \tag{8}$$

in which A_{ij}, B_{ij} and D_{ij} are the laminate stiffness coefficients as defined in the CLT with respect to the n- and t-axes, and N_j and M_j, are, respectively, the force and moment resultants per unit width induced by layer removal.

For a regular angle-ply laminate, containing only thermal shear stresses and no normal stresses, the force and moment resultants induced by layer removal will be N_{xy} and M_{xy} in the x, y co-ordinate system. They can be transformed to the N_i and M_i in the n, t co-ordinate system. For a non-regular angle-ply laminate or a general laminate with 0°, 90° and $\pm\varphi°$ layers, containing normal and shear stresses, the force and moment resultants induced by layer removal will be N_x, N_y, N_{xy}, M_x, M_y and M_{xy} in the x, y co-ordinate system, which again have to be transformed to the n, t co-ordinate system.

A_{nn} in Equation 8 is normally indicated by A_{11} in the CLT and for example A_{nnt} is A_{16}. Note that this is not true for σ_i, ε_i, N_i or M_i. For example $\sigma_1 \neq \sigma_n$.

Substitution of Equations 5 and 6 into Equation 8 gives an expression for the potential energy in terms of the unknown Ritz-coefficients. To minimize the potential energy with respect to the r_i, a set of eight equations can be formulated

$$\frac{\partial U}{\partial r_i}(r_i) = 0 \quad \text{with } r_i = a, a_1, a_3, b, b_1, b_3, c, d \tag{9}$$

In Equation 9, eight unknown Ritz-coefficients are present, namely a, a_1, a_3, b, b_1, b_3, c and d and the unknown force and moment resultants per unit width. Depending on the kind of laminate, for a regular angle-ply laminate, the unknown force and moment resultants are N_{xy} and M_{xy}; for a non-regular angle-ply laminate or a general laminate with 0°, 90° and $\pm\varphi°$ layers, they are N_x, N_y, N_{xy}, M_x, M_y, and M_{xy}.

To solve Equation 9 for a regular angle-ply laminate, at least two unknown parameters will have to be determined from measurements. These can be the curvatures in n- and t-directions represented by a and b. The curvatures a and b can be found from measuring the twisting and normal curvatures in the body axes directions and transforming them to the principal axes of curvature as explained before. To solve Equation 9 for the non-regular angle-ply laminate or the laminate with 0°, 90° and $\pm\varphi°$ layers, at least six unknowns will have to be measured. These then are three curvatures and the three strains.

Subsequently, Equation 9 can be solved to yield M_{xy}. Computing this for every layer removed, results in a twisting moment resultant per unit width profile against thickness removed. This result can be used in Equation 2 to calculate the shear stress profile in the original sample.

4. Determination of stresses, and strains and curvatures

The major contribution to residual stresses in composites are the thermal anisotropy stresses as indicated in Section 1. For validation purposes, they are now assumed to be the only stresses present. In particular, angle-ply laminates are used in which residual shear stresses are present. These residual shear stresses in the validation samples are calculated as a reference, using the MPE approach.

Also, the curvature and strain profiles, which would have been obtained by layer removal experiments on these validation samples are determined, from which the residual shear stresses are computed using the modified layer-removal analysis, accounting for large deformations and varying material properties through the thickness.

4.1. Thermal anisotropy stresses

For a symmetric angle-ply laminate, the thermal anisotropy stresses can be calculated using the CLT. However for asymmetric laminates, where out-of-plane deformations can be large, the energy approach described earlier must be used. This energy approach gives the same results as the CLT for small or no out-of-plane deflections, and therefore will also be used for calculations on symmetrical samples.

Equation 8 provides the basis for the calculations by the energy approach of the thermal curvatures and strains in laminates, resulting from cooling the laminates from the processing temperature to room temperature. During cooling, stresses and deformations will start to develop from the stress-free temperature, which is often the glass temperature for amorphous thermoplastic resins and is generally lower than the processing (melt) temperature.

For the calculation of the thermal strains and curvatures, only N_i and M_i in Equation 8, induced by layer removal, have to be replaced by thermally induced force and moment resultants N_i^T and M_i^T. N_i^T and M_i^T are given by

$$N_i^T = \sum_{f=1}^{n} \left(Q^*_{ij,f}(h_f - h_{f-1}) \int \alpha_{j,f} \mathrm{d}T \right)$$
$$M_i^T = \sum_{f=1}^{n} \left(Q^*_{ij,f}(h_f^2 - h_{f-1}^2) \int \alpha_{j,f} \mathrm{d}T \right) \qquad (10)$$

where n is the number of plies in the laminate, h_f is the thickness co-ordinate of the top of ply f, $\alpha_{i,f}$ are the thermal expansion coefficients of ply f in n-, t- and nt-components and $\mathrm{d}T$ is the temperature difference between stress-free temperature and room temperature.

The thermal anisotropy stresses in each ply f can subsequently be obtained from the unconstrained thermal strains ($\varepsilon^{\mathrm{T}}_{i,f} = \int \alpha_{i,f} \mathrm{d}T$) for each individual ply f and the thermal strains of the complete laminate ($\varepsilon^{\mathrm{T}}_{i,\mathrm{L}}(z) = \varepsilon^{0\,\mathrm{T}}_{i,\mathrm{L}} + z\,\kappa^{\mathrm{T}}_{i,\mathrm{L}}$) according to

$$\sigma(z)^{\mathrm{T}}_{i,f} = Q^*_{ij,f}(\varepsilon^{\mathrm{T}}_{i,\mathrm{L}}(z) - \varepsilon^{\mathrm{T}}_{i,f}) \qquad (11)$$

where subscript L indicates the laminate.

4.2. Curvature and strain profiles resulting from layer removal

Layer removal on an initially symmetric angle-ply laminate containing thermal anisotropy shear stresses, means that an asymmetric laminate is created which develops curvatures and strains. These curvatures (Fig. 3a) are equivalent to the curvatures obtained by directly cooling such an asymmetric laminate from stress-free temperature to room temperature (Fig. 3b). However, the strains are not the same. The strains in Fig. 3b are equal to the strains in Fig. 3a plus the initial strains in the original symmetric sample from which no layer is yet removed. This initial strain should therefore be subtracted from the strains determined in Fig. 3b.

5. Materials and validation examples

Theoretical validation experiments were performed on laminates from two different materials. The first material was a polyetherimide (PEI) resin reinforced with 50% continuous glass fibre. This material is typically used in compression moulding of plates, where angle-ply lay-ups are often used. The second material is polycarbonate (PC) resin reinforced with 40% short glass fibre. This material is used in injection moulding. Injection-moulded plates can show differences in orientation of the fibres in the plane of the moulding through the thickness [13–16], where the fibres are aligned in different layers at different angles, φ_i with the reference direction of a sample. The alignment of the fibres is caused by the mould geometry, the flow in the mould and the type of gating in the mould.

The required physical and mechanical properties of unidirectional plies made of these two materials are given in Table I. The unidirectional plies of the two

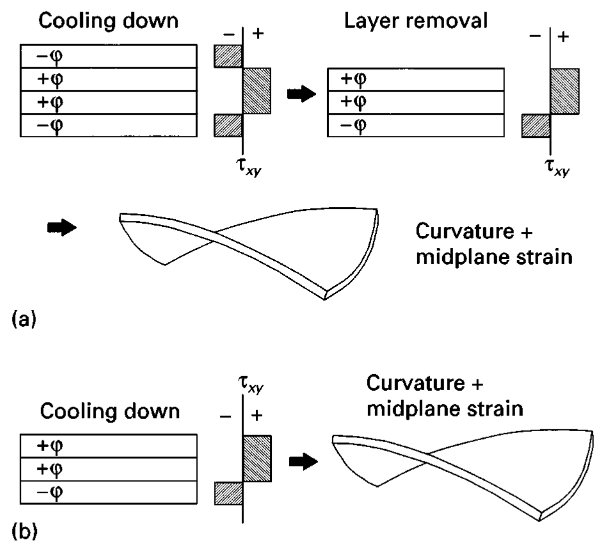

Figure 3 Comparison between deformations caused by (a) layer removal and (b) in directly cooling an asymmetric angle-ply laminate.

TABLE I Material properties of unidirectional plies used in the validation

Material (glass-content in volume percentage)	E_1 (GPa)	E^2 (GPa)	ν_{12}	G_{12} (GPa)	α_1 $(10^{-6}\,K^{-1})$	α_2 $(10^{-6}\,K^{-1})$	T_g (°C)
PEI/50% glass continuous	43	14.3	0.27	5.5	8	22	215
PC/18% glass short	11.2	3.6	0.32	1.2	12	69	150

materials were used in four different laminates. These laminates were used in the validation to examine the influence of the material and the sample size on the stresses calculated in the layer-removal analysis:

1. $(-30, 30)_{sym}$ PEI/glass, dimensions $90\ mm \times 30\ mm \times 2\ mm$, layer thicknesses $(0.5, 0.5)_{sym}$, ID code: pg93-ap;
2. $(-30, 30)_{sym}$ PEI/glass, dimensions $100\ mm \times 100\ mm \times 2\ mm$, layer thicknesses $(0.5, 0.5)_{sym}$, ID code: pg11-ap;
3. $(-30, 30)_{sym}$ PC/glass, dimensions $90\ mm \times 30\ mm \times 2\ mm$, layer thicknesses $(0.5, 0.5)_{sym}$, ID code: cg93-ap;
4. $(-30, 30)_{sym}$ PC/glass, dimensions $90\ mm \times 30\ mm \times 2\ mm$, layer thicknesses $(0.7, 0.3)_{sym}$, ID code: cg93-nap.

In addition, the influence of incorrectly neglecting the variation in mechanical properties over the thickness in the layer removal analysis of the shear stresses is examined. A typical modulus value of 30 GPa and Poisson's ratio of 0.3 were used for this incorrect analysis on sample pg93-ap.

6. Results

6.1. Reference stresses for laminates 1–4

The thermal anisotropy reference stresses in the chosen four laminates, were calculated according to the MPE approach using Equation 11. The results are given in Fig. 4. The results are the same for the two PEI/glass angle-ply laminates. The only difference between them is the size of samples, which has an influence only when curvature develops. Only half the laminate thickness is shown, because the stress profiles are symmetric with respect to the midplane.

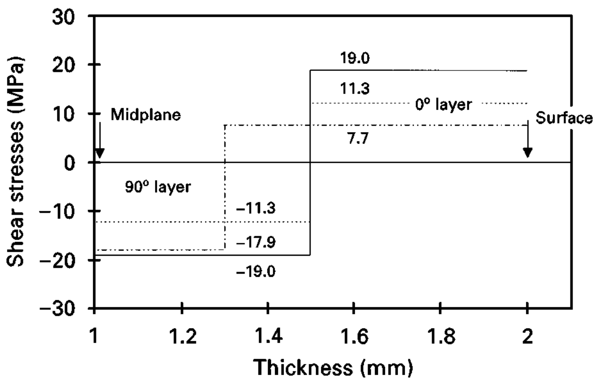

Figure 4 Thermal anisotropy reference stresses in laminates 1–4. (——) pg93/pg11-ap, (- - -) cg93-ap, (— - - —) cg93-nap.

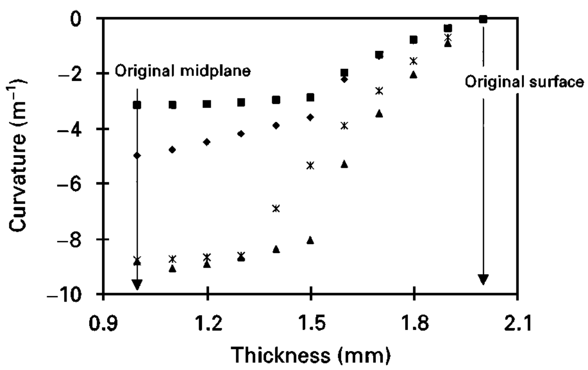

Figure 5 Thermal twisting curvature profiles (simulating layer-removal results). κ_{xy}: (◆) pg93-ap, (■) pg11-ap, (▲) cg93-ap, (∗) cg93-nap.

6.2. Curvature and strain profiles after layer removal

The curvature and strain profiles resulting from layer-removal experiments can be predicted theoretically by the MPE as the thermal curvatures and strains. These are calculated for the laminates, that would have been obtained by successive layer removal of thin layers from the originally symmetric validation laminates. Layer removal is done up to half the laminate thickness. Further layer removal is not performed because in these examples, the stress profile is assumed to be symmetric with respect to the midplane of the original laminate.

The thermal midplane strains in the original symmetric laminate (curvatures are zero) are subtracted from the thermal strains in laminates from which layers are removed for reasons explained earlier.

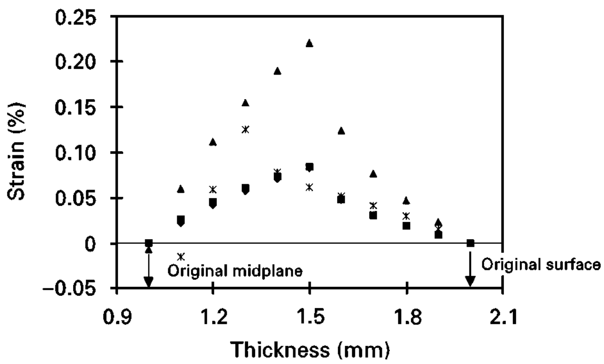

Figure 6 Thermal midplane shear strain profiles (simulating layer-removal results). ε^0_{xy}: (◆) pg93-ap, (■) pg11-ap, (▲) cg93-ap, (∗) cg93-nap.

The calculated thermal twisting curvature profiles and thermal midplane shear strain profiles are shown, respectively, in Figs 5 and 6. Also the normal curvatures and strains were calculated but not displayed.

6.3. Moments

From the curvature and strain profiles, the twisting moment profiles as a function of remaining laminate thickness can be calculated using the CLT, Equation 3. For the MPE approach, Equation 8, first the determined curvatures and strains are rotated towards the principal axes of curvature, where the twisting curvature is zero. The rotation angle is different for every separate laminate obtained by layer removal. Results are shown in Figs 7–10. Least-squares fitting was performed on the twisting moment values, which was used to calculate the shear stresses. Within a layer of constant fibre orientation, a straight line is fitted. This will result in a constant stress within the layer. The fitting is not performed over the interface to a next layer, because the stresses are discontinuous over the interface.

The moments were also calculated for the pg93-ap sample (PEI/glass) by the MPE approach, neglecting the variation in material properties. The next section shows the corresponding influence on the calculated shear stresses.

6.4. Shear stresses

The shear stresses were calculated from the twisting moment profiles using Equation 2 and a linear fit through the twisting moment points as indicated in the figures. These calculations were performed according to the CLT and the MPE. The MPE calculations give the same results as the residual reference shear-stress calculations in the original symmetric samples. Table II shows the results for the four different angle-ply laminates. Table II also shows the result for the pg93-ap angle-ply laminate when the material properties are taken to be constant over the thickness.

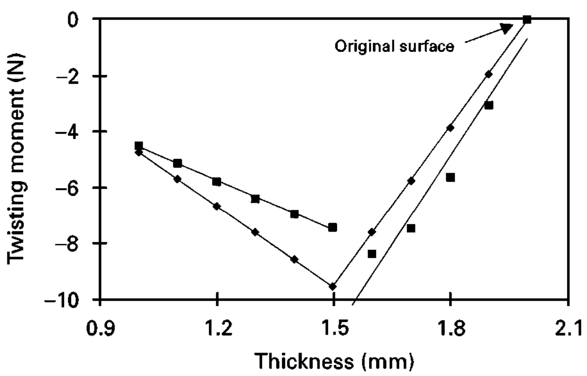

Figure 7 Twisting moment development after layer removal for pg93-ap sample (PEI/glass). M_{xy}: (◆) MPE, (■) CLT.

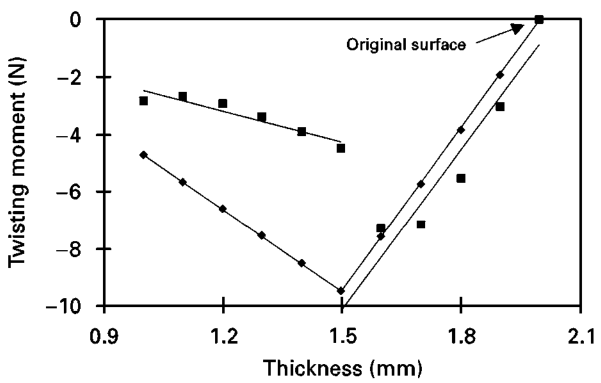

Figure 8 Twisting moment development after layer removal for pg11-ap sample (PEI/glass). M_{xy}: (◆) MPE, (■) CLT.

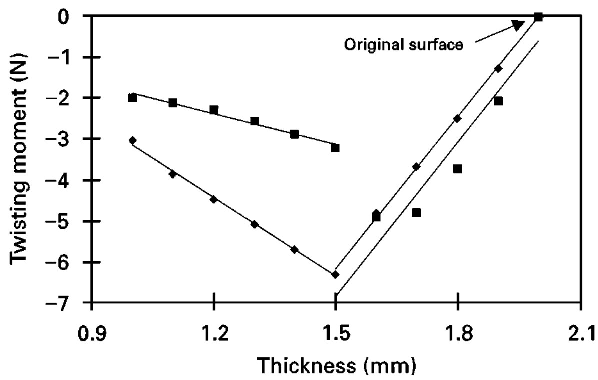

Figure 9 Twisting moment development after layer removal for cg93-ap sample (PC/glass). M_{xy}: (◆) MPE, (■) CLT.

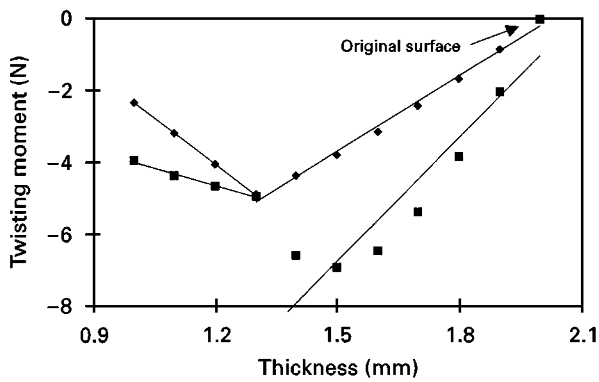

Figure 10 Twisting moment development after layer removal for cg93-nap sample (PC/glass). M_{xy}: (◆) MPE, (■) CLT.

TABLE II Shear stresses and errors from layer-removal analysis validation for different examples

Sample	σ_{xy}(− 30°-layer)[a]			
	Surface		Core	
pg93-ap (MPE)	18.9	(− 1)	− 19.1	(+ 1)
pg93-ap (CLT)	20.8	(+ 9)	− 12.3	(− 35)
pg1010-ap (MPE)	18.8	(− 1)	− 18.8	(− 1)
pg1010-ap (CLT)	18.2	(− 4)	− 6.5	(− 66)
cg93-ap (MPE)	12.4	(+ 2)	− 13.0	(+ 7)
cg93-ap (CLT)	12.3	(+ 1)	− 4.7	(− 61)
cg93-nap (MPE)	6.8	(− 12)	− 16.8	(− 6)
cg93-nap (CT)	11.0	(+ 43)	− 7.0	(− 61)
pg93-ap, constant properties	22.4	(+ 18)	− 14	(− 26)

[a] Percent error given in parenthesis.

7. Discussion

The modified layer-removal analysis using the MPE approach leads to the reference shear stresses for the three laminates. The small errors occurring are caused by the angle transformation in the analysis to a coordinate system with no shear strain and twisting curvature. This angle is determined using curvatures calculated according to the CLT, which gives only an approximation of the needed rotation angle.

The errors in the stresses according to the CLT calculations arise because the twisting moments are not calculated correctly. Therefore, also the slope of the twisting moment lines, as shown in Figs 7–10, will not be correct. They are both present in Equation 2 from which the stresses are calculated. Further, the twisting moment development should be linear, resulting in constant shear stresses. However, Figs 7–10 show clearly a non-linear moment development. Performing a higher order polynomial fit in this case gives a better result, but leads to significant stress variation where the stresses should be constant.

Table II reveals several important points. The pg93-ap sample shows a large error in the stress value of the core layer using the CLT calculation. In the surface layer the error is negligible. The pg11-ap, where the sample dimensions are increased to 100 mm × 100 mm, shows a larger error in the core layer. In the surface layer the error is small. The errors in the core layers are large, because the geometric non-linear laminate behaviour is the most pronounced because of the larger curvatures which occur. In the surface layers, the curvatures are smaller and CLT calculation will therefore result in smaller errors. The differences in errors between the pg93-ap sample and the pg11-ap sample is caused by a size effect. This effect of the sample dimensions can be illustrated with a diagram of the thermal curvatures calculated by the MPE, in principal curvature axes in a square composite angle-ply plate after manufacture as a function of the side length of the plate. Such diagrams can be found in the literature [7–10] (Fig. 11). In Fig. 11, the result of the CLT calculation is given by the curvatures at side length zero. The bifurcation point indicates the side length where the anticlastic shape of the laminate changes to a cylindrical shape. In this situation, only one curvature is present and the other is suppressed.

Fig. 11 clearly shows that for very small side lengths, the CLT and MPE give the same results for the curvatures in n- and t-directions. For larger side lengths, the error between CLT and MPE can be large. This implies that the sample dimension choice should not be arbitrary. For ease of analysis, it is advisable to choose such sample dimensions, so that the linear CLT calculation approximates the MPE calculation. However, this might not always be possible, due to practical limitations.

The results of the cg93-ap sample (PC/glass) show that for this specific laminate the geometric non-linear behaviour in the layer-removal experiments is important. The errors in the calculation according to the CLT are similar to those for the PEI/glass samples. Further, the errors in the calculations on the non-regular cg93-nap sample in which the layer thicknesses are changed, are even larger. They cannot be neglected in the layer-removal analysis. This laminate has material properties which are very similar to those of injection-moulding materials. So, in principle, such analysis can be performed on injection-moulded short fibre-reinforced products.

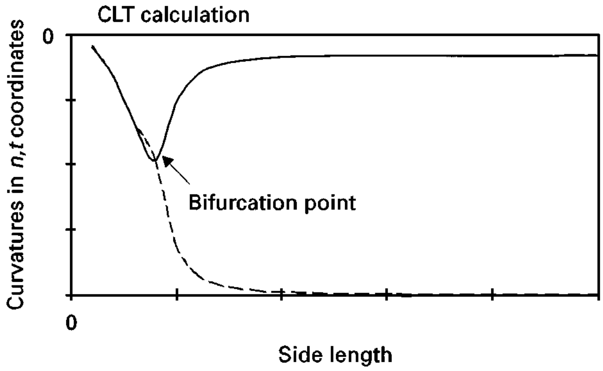

Figure 11 Schematic diagram of the thermal curvatures in a sequence composite angle-ply plate as function of side length. (——) Predicted n-curvature, (– – –) predicted negative t-curvature.

Table II further shows the influence of approximating the varying material properties by constant material properties. The errors introduced in a CLT calculation are large for the continuous fibre-reinforced PEI/glass sample. Calculations according to the MPE approach yielded values which were off by more than 100%. Also, but not illustrated in this paper, the typical injection-moulded sample PC/glass showed similar errors in the CLT calculation when the material properties were assumed constant (E-modulus 9 GPa and Poisson's ratio 0.3).

8. Conclusion

The layer-removal method is further developed for use on composite materials containing shear stresses and with varying material properties through the thickness. The method produced good theoretical results in the determination of residual shear stresses for a typical injection-moulded plate and a typical compression-moulded plate. Also, the need to use the variation in material properties is clearly demonstrated. Using constant material properties in the analysis of the layer-removal results, leads to large errors in calculated shear stresses.

The deformations in layer-removal experiments on composite materials can become large and the CLT can no longer be used for a correct evaluation of the layer-removal results. Instead, the MPE approach described in this paper can be used for a correct calculation of the stresses from layer-removal experiments. In these calculations, only curvatures have to be measured for laminates containing only residual shear stresses. Curvatures as well as midplane strains are needed and have to be measured for laminates containing residual shear and normal stresses. Neglecting strains or curvatures leads, in general, to unacceptable errors. The CLT provides, only in specific cases, a good approximation of the moments induced by layer removal, leading to small errors in stress values.

References

1. K. S. KIM, H. T. HAHN and R. B. CROMAN, *J. Compos. Technol. Res.* **11** (1989) 47.
2. G. JERONIMIDIS and A. T. PARKYN, *J. Compos. Mater.* **22** (1988) 401.
3. R. G. TREUTING and W. T. READ Jr, *J. Appl. Phys.* **22** (1951) 130.
4. J. R. WHITE, *J. Mater. Sci.* **20** (1985) 2377.
5. M. W. A. PATERSON and J. R. WHITE, *ibid.* **24** (1989) 3521.
6. M. W. HYER, *J. Compos. Mater.* **15** (1981) 175.
7. *Idem*, *ibid.* **15** (1981) 296.
8. W. J. JUN and C. S. HONG, *Compos. Sci. Technol.* **38** (1990) 55.
9. *Idem*, *J. Reinf. Plast. Compos.* **11** (1992) 1352.
10. L. J. B. PEETERS, P. C. POWELL and L. L. WARNET, *J. Compos. Mater.* **30** (1996) 603.
11. P. C. POWELL, in "Engineering with Fibre-Polymer Laminates" (Chapman and Hall, London, 1994).
12. R. T. FENNER, in "Engineering Elasticity" (Ellis Horwood, Chichester, 1986).
13. H. BIJSTERBOSCH and R. J. GAYMANS, *Polym. Compos.* **16** (1995) 363.
14. S. KENIG, *ibid.* **7** (1986) 50.
15. M. W. DARLINGTON and A. C. SMITH, *Polym. Compos.* **8** (1987) 16.
16. R. S. BAY, C. L. TUCKER III and R. B. DAVIS, *ANTEC Technic, Papers* **35** (1989) 539.

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